

**BMath Algebra-I
Backpaper Exam
2016-2017**

Time: 3 hrs
Max score: 100

Answer all questions.

- (1) (a) Let G be a group and H be a subgroup of G . Consider the action of G on the left cosets of H in G by left multiplication. Determine the kernel of the action and show that the kernel is the largest normal subgroup of G contained in H .
(b) Prove that if H has finite index n then there is a normal subgroup K of G , $K \subseteq H$, such that $|G : K| \leq n!$. (8+6)
- (2) (a) Determine class equation for a finite group G .
(b) Show that if $o(G) = p^n$ for some prime p and some positive integer n , then the center of G is non-trivial. (8+6)
- (3) (a) Prove that two elements of S_n are conjugate if and only if they have the same cycle type.
(b) Determine the elements of $C_{S_7}(\sigma)$ where $\sigma = (1\ 4\ 5)$. (8+8)
- (4) (a) State the three Sylow's theorems.
(b) Show that a group of order p^2q , where p and q are distinct primes, is not simple. (8+8)
- (5) Calculate the number of Sylow p -subgroups of
(a) A_5 , and
(b) S_5 . (7+7)
- (6) (a) Define the semi-direct product of two groups.
(b) Show that D_{2n} , the dihedral group of order $2n$, is the semi-direct product of the cyclic groups Z_n and Z_2 . (4+8)
- (7) Classify groups of order 12 whose Sylow 3-subgroup is normal. (14)

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